**Investments Problem Sheet 4 Lent Term 2024**

**True-False**

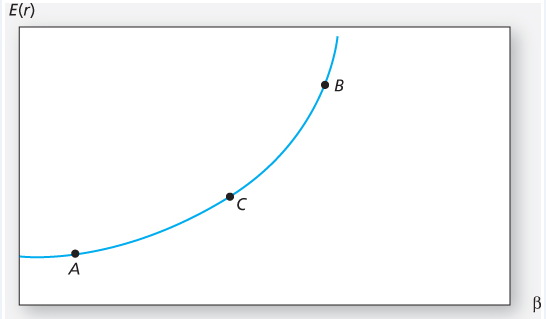
1. Arbitrage Pricing Theory says that if there is no arbitrage then all share prices must be driven by a small number of factors.

**False. The APT *assumes* that all share prices are driven by a small number of factors.**

1. If a substantial body of investors decided to exclude “sin stocks” (eg tobacco, armaments) from their portfolios, both the Arbitrage Pricing Theory (APT) and the Capital Asset Pricing Model (CAPM) would continue to hold.

**False. The CAPM would not hold. If some investors exclude sin stocks, their price would drop, their expected return would rise, and unrestricted investors would find it worth buying more sin stocks. The market would no longer be efficient; the efficient portfolio would be overweight sin stocks. The CAPM would only hold if the market portfolio is interpreted as the portfolio held by unrestricted investors. APT would continue to hold but there is likely to be a priced “sin” factor.**

1. If expected return rises with beta as shown in the following figure, there is a clear arbitrage opportunity.



**True.** **A long position in a portfolio (P) comprised of Portfolios A and B will offer an expected return-beta tradeoff lying on a straight line between points A and B. Therefore, we can choose weights such that βP = βC but with expected return higher than that of Portfolio C. Hence, combining P with a short position in C will create an arbitrage portfolio with zero investment, zero beta, and positive rate of return.**

4. Consider a single factor APT. Portfolio A has a beta of 1.0 and an expected return of 12%. Portfolio B has a beta of 1.5 and an expected return of 17%. The risk-free rate of return is 4%. If you wanted to take advantage of an arbitrage opportunity, you should take a short position in portfolio A and a long position in portfolio B.

**TRUE. A: 12% = 1.0F + 4%; F = 8%; B: 17% = 1.5F + 4%: F = 8.67%; thus, short A and take a long position in B.**

**Questions**

1. You are working for an investment house that believes in the APT. They use a four factor model, as in Chen, Roll and Ross. They have constructed four portfolios that are maximally correlated with each of the four factors (unexpected changes in inflation, slope of the term structure, corporate risk premium and output). They have estimated factor betas for a number of shares, and you are particularly interested in three of the shares (A, B and C). Their betas are given below:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Share | Factor1 | Factor 2 | Factor 3 | Factor 4 |
| A | +0.5 | +1.4 | -0.2 | +1.0 |
| B | -0.8 | +2.0 | +0.5 | +0.7 |
| C | +3.1 | +0.2 | -1.6 | +1.6 |

1. Given that the risk free rate is 5% and the risk premium on the four factors is estimated at 2.0%, 1.0%, 1.5% and 1.0%, what is the expected return on each of the shares?

**On A the expected return is 5% + 0.5x2.0% + 1.4x1.0% - 0.2x1.5% + 1.0x1.0% =8.1%. On B it is 5% - 0.8x2.0% + 2.0x1.0% + 0.5x1.5% + 0.7x1.0% =6.85%, while on C it is 5% +3.1x2.0% + 0.2x1.0% - 1.6x1.5% + 1.6x1.0% =10.6%.**

1. What are the betas of a portfolio that is invested two thirds in B and one third in C? What is its expected return? How does the portfolio differ from one that is invested fully in A?

**The betas are {(-2x0.8 + 3.1)/3, (2x2.0+0.2)/3, (2x0.5-1.6)/3, (2x0.7+1.6)/3}. These are {0.5, 1.4, -0.2, 1.0} which is the same as A. The expected return must be the same as A, at 8.1%. The difference between the portfolio and A is idiosyncratic risk – risk that is uncorrelated with any of the four factors, and which attracts no expected return.**

2. Suppose there are two independent factors F1 and F2 . All stocks have independent firm-specific components with a standard deviation of 45%. The following are well-diversified portfolios.

|  |  |  |  |
| --- | --- | --- | --- |
| Portfolio | Beta on F1 | Beta on F2 | Expected Return |
| A | 1.5 | 2 | 31% |
| B | 2.2 | -0.4 | 26% |
| C | 0 | 0 | 6% |

What is the expected return-beta relationship in this economy?

Solution:

Portfolio C has zero betas.

Hence, its return must be the risk-free rate of return = 6%

E(rp ) = rf + βP1 [E(r1 ) − rf ] + βP2 [E(r2 ) – rf ]

We need to find the risk premium (RP) for each of the two factors:

RP1 = [E(r1 ) − rf ] and RP2 = [E(r2 ) − rf ]

In order to do so, we solve the following system of two equations with two unknowns:

.31 = .06 + (1.5 × RP1 ) + (2.0 × RP2 )

.26 = .06 + (2.2 × RP1 ) + [(–0.4) × RP2 ]

The solution to this set of equations is:

RP1 = 10% and RP2 = 5%

Thus, the expected return-beta relationship is:

E(rP ) = 6% + (βP1 × 10%) + (βP2 × 5%)

3. The following annual excess rates of return were obtained for nine individual stocks and a market index.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year | Market Excess Returns(%) | A | B | C | D | E | F | G | H | I |
| 1 | 29.65 | 33.88 | -25.20 | 36.48 | 42.89 | -39.89 | 39.67 | 74.57 | 40.22 | 90.19 |
| 2 | -11.91 | -49.87 | 24.70 | -25.11 | -54.39 | 44.92 | -54.33 | -79.76 | -71.58 | -26.64 |
| 3 | 14.73 | 65.14 | -25.04 | 18.91 | -39.86 | -3.91 | -5.69 | 26.73 | 14.49 | 18.14 |
| 4 | 27.68 | 14.46 | -38.64 | -23.31 | -0.72 | -3.21 | 92.39 | -3.82 | 13.74 | 0.09 |
| 5 | 5.18 | 15.67 | 61.93 | 63.95 | -32.82 | 44.26 | -42.96 | 101.67 | 24.24 | 8.98 |
| 6 | 25.97 | -32.17 | 44.94 | -19.56 | 69.42 | 90.43 | 76.72 | 1.72 | 77.22 | 72.38 |
| 7 | 10.64 | -31.55 | -74.65 | 50.18 | 74.52 | 15.38 | 21.95 | -43.95 | -13.40 | 28.95 |
| 8 | 1.02 | -23.79 | 47.02 | -42.28 | 28.61 | -17.64 | 28.83 | 98.01 | 28.12 | 39.41 |
| 9 | 18.82 | -4.59 | 28.69 | -0.54 | 2.32 | 42.36 | 18.93 | -2.45 | 37.65 | 94.67 |
| 10 | 23.92 | -8.03 | 48.61 | 23.65 | 26.26 | -3.65 | 23.31 | 15.36 | 80.59 | 52.51 |
| 11 | -41.61 | 78.22 | -85.02 | -0.79 | -68.70 | -85.71 | -45.64 | 2.27 | -72.47 | -80.26 |
| 12 | -6.64 | 4.75 | 42.95 | -48.60 | 26.27 | 13.24 | -34.34 | -54.47 | -1.50 | -24.46 |

Suppose that in addition to the market factor that has been considered, a second factor is considered. The values of this factor for years1 to 12 were as follows:

|  |  |
| --- | --- |
| Year | % Change in Factor Value |
| 1 | -9.84 |
| 2 | 6.46 |
| 3 | 16.12 |
| 4 | -16.51 |
| 5 | 17.82 |
| 6 | -13.31 |
| 7 | -3.52 |
| 8 | 8.43 |
| 9 | 8.23 |
| 10 | 7.06 |
| 11 | -15.74 |
| 12 | 2.03 |

1. Perform the first-stage time-series regressions and tabulate the relevant summary statistics (Hints: use a multiple regression as in a standard spreadsheet package. Estimate the betas of the 12 stocks on the two factors).

**Answer:**

The first-stage time-series (SCL) regression results are summarized below:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *R*-square | A | B | C | D | E | F | G | H | I |
| 0.07 | 0.36 | 0.11 | 0.44 | 0.24 | 0.84 | 0.12 | 0.68 | 0.71 |
| Observations | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| Intercept | 9.19 | -1.89 | -1.00 | -4.48 | 0.17 | -3.47 | 5.32 | -2.64 | 5.66 |
| Beta *M* | -0.47 | 0.58 | 0.41 | 1.39 | 0.89 | 1.79 | 0.65 | 1.91 | 2.08 |
| Beta *F* | -0.35 | 2.33 | 0.67 | -1.05 | 1.03 | -1.95 | 1.15 | 0.43 | 0.48 |
| *t*-intercept | 0.71 | -0.13 | -0.08 | -0.37 | 0.01 | -0.52 | 0.29 | -0.28 | 0.59 |
| *t*-Beta *M* | -0.77 | 0.87 | 0.75 | 2.46 | 1.40 | 5.80 | 0.75 | 4.35 | 4.65 |
| *t*-Beta *F* | -0.34 | 2.06 | 0.71 | -1.08 | 0.94 | -3.69 | 0.77 | 0.57 | 0.63 |

1. Specify the hypothesis for a test of a second-stage regression for the two-factor Security Market Line.

**Answer:**

The hypotheses for the second-stage cross-sectional regression for the two-factor SML are:

* The intercept is zero.
* The market-index slope coefficient equals the market-index average return.
* The factor slope coefficient equals the average return on the factor.(Note that the first two hypotheses are the same as those for the single factor model.)

1. Do the data suggest a two-factor economy?

Answer:

The inputs for the second-stage cross-sectional regression are:

|  |  |  |  |
| --- | --- | --- | --- |
| A | Average Excess Return | Beta M | Beta F |
| 5.18 | -0.47 | -0.35 |
| B | 4.19 | 0.58 | 2.33 |
| C | 2.75 | 0.41 | 0.67 |
| D | 6.15 | 1.39 | -1.05 |
| E | 8.05 | 0.89 | 1.03 |
| F | 9.90 | 1.79 | -1.95 |
| G | 11.32 | 0.65 | 1.15 |
| H | 13.11 | 1.91 | 0.43 |
| I  M | 22.83 | 2.08 | 0.48 |
| 8.12 |  |  |
| F | 0.60 |  |  |

The second-stage cross-sectional regression yields:

Regression Statistics

Multiple R 0.7234

R-square 0.5233

Adjusted R-square 0.3644

Standard error 4.87 Observations 9

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Coefficients | Standard Error | *t* Statistic for β =0 | *t* Statistic for β *t* Statistic for β  =8.12 =0.6 |
| Intercept | 3.35 | 2.88 | 1.16 |  |
| Beta M | 5.53 | 2.16 | 2.56 | -1.20 |
| Beta F | 0.80 | 1.42 | 0.56 | 0.14 |

These results are slightly better than those for the single factor test; that is, the intercept is smaller and the slope of M is slightly greater. We cannot expect a great improvement since the factor we added does not appear to carry a large risk premium (average excess return is less than 1%), and its effect on mean returns is therefore small. The data do not reject the second factor because the slope is close to the average excess return and the difference is less than one standard error. However, with this sample size, the power of this test is extremely low.